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Coulombic effects on fermion masses in models with standard model fields in large extra dimensions

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Abstract

We study the effects of Coulombic interactions between fermions in generic models with large extra dimensions in which standard model fields propagate. It is suggested that these interactions could help to explain (i) why the heaviest known fermion is a charge $2/3$ quark, rather than a charge $-1/3$ quark or a lepton, (ii) why this fermion has a mass m_t comparable to the electroweak symmetry breaking scale M_{ew} and, (iii) the patterns $m_t \gg m_b > m_\tau$ and $m_c \gg m_s > m_\mu$.

The explanation of the spectrum of quark and charged lepton masses and the Cabibbo–Kobayashi–Maskawa (CKM) quark mixing matrix is an outstanding challenge that requires physics beyond the standard model (SM). There are several general features that one would like to understand. Why is the heaviest known fermion a charge $2/3$ quark, rather than a charge $-1/3$ quark or a lepton? Why does this heaviest known fermion, the top quark, have a mass that is comparable to the electroweak symmetry breaking (EWSB) scale $M_{\mathrm{ew}} = v/\sqrt{2} = 174$ GeV, where $v = 2^{-1/4}G_F^{-1/2} = 246$ GeV? Why, in each generation, are the quarks heavier than the leptons and why, in the heavier two generations, is the mass of the charge $2/3$ quark greater than the mass of the charge $-1/3$ quark?

Recently, a new approach to fermion mass hierarchies has been considered, in which one assumes an underlying higher-dimensional spacetime and obtains the hierarchies from the localization of fermion wavefunctions at different points in the higher dimensions [1,2]. Here we study effects of Coulombic gauge interactions between fermions in this type of theory. We show that these effects are important and could help to explain the above-mentioned features of fermion masses. We use the notation u_i, d_i, and e_i with $i = 1, 2, 3$ to refer, respectively, to u, c, t, d, s, b, and e, μ, τ.

Let us briefly describe the framework. For each generation, we denote the left-handed fermion fields as Q_i and L_i for the quark and lepton $SU(2)$ doublets and u_i^c, d_i^c, and e_{iL}^c for the $SU(2)$ singlets. Usual spacetime coordinates are denoted as x_ν, $\nu = 0, 1, 2, 3$, and the n extra coordinates as y_λ; for definiteness, the latter are taken to be compact. Generic fermion fields are denoted $\Psi(x, y) = \psi(x)\chi(y)$. In the extra dimensions

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the gauge, Higgs, and fermion fields are assumed to have support in an interval $0 \leqslant y_\lambda \leqslant L$ [3]. The $d = (4+n)$-dimensional fields thus have Kaluza–Klein (KK) mode decompositions. We shall work in a low-energy effective field theory approach. The gauge fields extend over the interval $0 < y_\lambda < L$, consistent with the observed universality of gauge-fermion couplings. A similar assumption is made for the Higgs field(s). The fermion wavefunctions are localized at different values of y_λ. Such localization could occur naturally in string theories [4]. Here, in the context of a low-energy effective field theory, one obtains this localization via interactions with a scalar field. For example, for the case $n = 1$, before the inclusion of gauge interactions, consider the action for the quarks:

$$S \propto \int d^4x\,dy \Big[\sum_i \overline{\Psi}_{Q_i}\big(i\not{\partial} + L^{1/2}\Phi - \mathcal{M}_{0,Q_i}\big)\Psi_{Q_i} + \sum_i \sum_{f=u,d} \overline{\Psi}_{f_i^c}\big(i\not{\partial} + L^{1/2}\Phi - \mathcal{M}_{0,f_i}\big)\Psi_{f_i^c} + L^{1/2}\sum_{i,j}\big(\kappa_{d,ij}\Psi^T_{Q_i}C_5\Psi_{d_j^c}H_d + \kappa_{u,ij}\Psi^T_{Q_i}C_5\Psi_{u_j^c}H_u + \text{h.c.}\big)\Big] \qquad (1)$$

where $\not{\partial}$ and C_5 are the five-dimensional Dirac operator and charge conjugation matrix. In (1), in the SM, $H_d = H^\dagger$ and $H_u = \widetilde{H}^\dagger$, with H being the SM Higgs and $\widetilde{H} = i\sigma_2 H^*$; more generally, H_u and H_d may be independent Higgs fields, as in supersymmetric extensions of the SM. A similar formula holds for leptons. The proportionality factor in (1) is chosen to yield a canonically normalized 4D action. If $\Phi(x,y)$ has the usual kink solution $\Phi = \Phi_0 \tanh(\mu y)$, this traps each fermion to a domain wall at $y_{f_i} = \ell_{f_i} = -\mathcal{M}_{0,f_i}/\mu^2$ [5,6] with localization length μ^{-1}. Starting with Dirac fermions in the five-dimensional space, this trapping mechanism yields a chiral theory in which only left-handed fermions are trapped on the physical 4D domain wall. In the standard model, the fermions trapped to the physical domain wall are then Q_i, u_i^c, d_i^c, L_i, and e_i^c, $i = 1,2,3$. More generally, one considers the possibility of fermion localization with $n > 1$; for $n = 2$ a vortex solution can provide fermion localization [5]. In the quark sector, for N_g generations, one can choose the values of the $n(3N_g - 1)$ coordinate differences for the wavefunction centers ℓ_{f_i} to fit the $2N_g$ quark masses and $(N_g - 1)^2$ parameters determining the CKM matrix, i.e., the $N_p = N_g^2 + 1$ physical parameters.

If $\mu^{-1} \ll L$; then, to a good approximation, a generic fermion wavefunction is a Gaussian of the form $\Psi(x,y) = \psi(x)\chi(y)$ with $\chi(y) \propto e^{-\mu^2(y-\ell)^2/2}$, where $(y-\ell)^2 \equiv \sum_{\lambda=1}^n (y_\lambda - \ell_\lambda)^2$. Performing the integration over y, one obtains the 4D quark Yukawa couplings

$$S_Y = \sum_{i,j} \kappa^{(4D)}_{d,ij} \int d^4x\, \bar{\psi}_{d_jR}(x)\psi_{Q_iL}(x)H_d(x) + \sum_{i,j} \kappa^{(4D)}_{u,ij} \int d^4x\, \bar{\psi}_{u_jR}(x)\psi_{Q_iL}(x)\widetilde{H}_u(x) + \text{h.c.}, \qquad (2)$$

where

$$\kappa^{(4D)}_{f,ij} = \exp\big(-\mu^2(\ell_{Q_i} - \ell_{f_j^c})^2/4\big)\kappa_{f,ij}. \qquad (3)$$

Thus, separations $|\ell_{Q_i} - \ell_{f_j^c}|$ that are moderate in units of μ^{-1} produce a strong Gaussian suppression of fermion overlaps and hence of the associated 4D Yukawa couplings. Similar comments apply for the charged leptons. Since a purpose of this type of model is to derive a hierarchy without putting it in initially, one takes the $\kappa_{f,ij} \sim O(1)$. By choosing the different separations, one can account for the observed fermion mass and mixing angle hierarchies in terms of the relative locations of fermions in the extra dimensions and further explain proton longevity and the weakness of flavor-changing neutral current processes. This type of model involves three general length scales: L, μ^{-1}, and, since it is a low-energy effective field theory, a high-energy cutoff, Λ. Besides the $\mu^{-1} \ll L$ condition, one requires $\mu \ll \Lambda$ for the self-consistency of the theory; typical ratios are $\mu/L^{-1} \simeq 20$–30 and $\Lambda/\mu \sim 20$ [2]. The coupling $\kappa^{(4D)}_{f,ij}$ depends on these length scales via the product $\mu|\ell_{Q_i} - \ell_{f_j^c}|$, and our results such as Eqs. (11) and (13) below depend on the dimensionless products $\mu d_{f,ij}$ (where $d_{f,ij}$ is a separation distance between fermions) and the ratio $L/d_{f,ij}$ but are not sensitively dependent on the value of L itself, given that is is accord with experimental constraints. The lower bound on L^{-1} depends on the type of model; for example, the value $L^{-1} \gtrsim 100$ TeV was used in [2] (for adequate suppression of neutral flavor-changing currents) [6].

We proceed to incorporate the SM $SU(3) \times SU(2) \times U(1)_Y$ gauge interactions. Our notation is indicated by the covariant derivative on quarks,

$$D_\mu = \partial_\mu - ig_3 C_\mu - ig_2 A_\mu P_L - i(g'/2)(Y_L P_L + Y_R P_R) B_\mu,$$

where

$$C_\mu = \sum_{a=1}^{8} C_\mu^a (\lambda_a/2), \qquad A_\mu = \sum_{a=1}^{3} A_\mu^a (\tau_a/2),$$

C_μ^a, A_μ^a and B_μ are the $SU(3)$ color, $SU(2)_L$, and weak hypercharge gauge bosons, and $P_{L,R}$ are chiral projection operators. Thus, $g'/g_2 = \tan\theta_W$ and $g' = (3/5)^{1/2} g_1$, where g_i, $i = 1, 2, 3$, are the couplings that would unify $(= g)$ at a high mass scale in a grand unified theory.[1]

Because the fermions are localized in the higher dimensions, with the wavefunction factorization given above, they are essentially static as functions of y, so we need only consider the Coulombic interaction between them. Since $\mu^{-1} \ll L$, the Coulomb potential is that for the full $(d = 4 + n)$-dimensional space. Also $\mu \gg L^{-1} \gg M_{\mathrm{ew}}$, so (i) the relevant gauge fields are massless on this scale, and (ii) the color contribution is perturbatively calculable to good accuracy. The Yukawa operators $\bar{d}_{jR} Q_{iL} H_d$ and $\bar{u}_{jR} Q_{iL} H_u$ yield, via the Higgs vevs, the bilinears $\bar{d}_{jR} d_{iL}$ and $\bar{u}_{jR} u_{iL}$. Each of these bilinears involves only the one $SU(2)$ nonsinglet fermion, Q_{iL}, so only the (vectorial) $SU(3)_c$ color and (chiral) $U(1)_Y$ hypercharge interactions between these fermions contribute. The distance between the centers of the wavefunctions of the fermions f_{iL} and f^c_{jL} at (x, ℓ_{f_i}) and $(x, \ell_{f^c_j})$ is $d_{f,ij} = |\ell_{f_i} - \ell_{f^c_j}|$. Using $Y_{Q_L} = 1/3$, $Y_{L_L} = -1$, $Y_{f_R} = 2Q_f$, the Coulomb interaction energy of f_{iL} and $\bar{f}_{jR}$ is

$$V_{\mathrm{Coul}}(y) = -\frac{a_f L^n}{|y|^{1+n}} \tag{4}$$

with $|y| = d_{f,ij}$, where $a_f = c_f / A_{3+n}$, $A_\ell = 2\pi^{\ell/2}/\Gamma(\ell/2)$ is the area of the unit sphere S^ℓ, and

$$c_u = \frac{4}{3} g_3^2 + \frac{4}{15} g_1^2, \tag{5}$$

$$c_d = \frac{4}{3} g_3^2 - \frac{2}{15} g_1^2, \tag{6}$$

$$c_e = \frac{6}{5} g_1^2. \tag{7}$$

For example, in the operator $\bar{u}_{jR} u_{iL}$ one has two fundamental representations of color $SU(3)$ contracted to a singlet, so the color interaction is attractive, and the coupling constant dependence is $-(4/3)g_3^2$.[2] Since the hypercharges Y of the u_{iL} and $\bar{u}_{jR}$ are $1/3$ and $-4/3$, the hypercharge interaction is $(1/3)(-4/3)(g')^2 = -(4/15)g_1^2$, as given above. The normalization in (4) satisfies the requirement that as $|y|$ increases through the value L beyond which the effective dimension of spacetime is 4, the potential matches the usual 4D Coulomb potential. Note the general inequality

$$c_u > c_d \geqslant c_e. \tag{8}$$

If one makes the additional assumption of gauge coupling unification $g_i = g$ at some scale M_U with $\mu > M_U \gtrsim L^{-1}$, then for distances d such that $\Lambda^{-1} < d < M_U^{-1}$, $c_u = (8/5)g^2$ and $c_d = c_e = (6/5)g^2$.

Before inclusion of gauge interactions, the suppression in (3) can be treated as the result of the quantum-mechanical tunnelling of each fermion to the midpoint of the line joining ℓ_{f_i} and $\ell_{f^c_j}$, located at a distance $d_{f,ij}/2$ from each [2]. The WKB tunnelling amplitude [7] for each fermion is $\exp(-\int_0^{d_{f,ij}/2} V(r)\,dr) = \exp(-(\mu d_{f,ij})^2/8)$, yielding, for the total suppression, the factor $\exp(-(\mu d_{f,ij})^2/4)$, in agreement with (3).

We now consider the effect of the Coulombic gauge interactions. Since $L/d_{f,ij}$ values are reasonably large, the effects of the boundary conditions on the gauge fields, e.g., image charges, are small, and we neglect them. Since the fermions have the same localization lengths, the tunnelling can be treated in a symmetric way for each pair. The full potential energy for f_i at a distance r outward from its wavefunction center toward f^c_j is $V_f(r) = V_{\mathrm{trap}}(r) + V_{\mathrm{Coul}}(d_{f,ij} - r)$, where $V_{\mathrm{trap}}(r) = \mu^2 r$. The resultant restoring force in

[1] Rapid power law running of gauge and Yukawa couplings occurs in the present models since $d > 4$. This can lead to precocious gauge coupling unification. However, the placement of fermions at different positions explicitly breaks both $SU(5)$ and $SO(10)$; e.g., as components of the 10_L of $SU(5)$, Q_i, u^c_i, and e^c_i would necessarily have wavefunctions centered at the same point if the underlying theory were invariant under $SU(5)$.

[2] For general N_c, the factor 4/3 is $C_2(\mathrm{fund.}) = (N_c^2 - 1)/(2N_c)$.

the $\hat{\mathbf{r}}$ direction is $F = -\mu^2 + a_f(n+1)L^n/(d_{f,ij} - r)^{n+1}$. The most dramatic effect occurs if $F(r = 0^+) = -\mu^2 + a_f(n+1)L^n/(d_{f,ij})^{n+2}$ is positive, i.e.,

$$(\mu d_{f,ij})^2 < a_f(n+1)\left(\frac{L}{d_{f,ij}}\right)^n, \tag{9}$$

here, the Coulomb attraction overwhelms the trapping potential and causes f_i and f_j^c to have wavefunctions that are centered at the same location, at least to within the distance Λ^{-1} down to which the low-energy effective field theory applies. Hence in this case there is no suppression of the Yukawa coupling $\kappa^{(4D)}_{f,ij}$, so that, if the higher-dimensional coupling $\kappa_{f,ij} \sim O(1)$, then the resultant fermion mass is of order M_{ew}. Let us first concentrate on the third generation and neglect small mixing effects. We further focus on the $n = 2$ case since a successful minimal fit was found for this case [2] with $\mu d_{u,33} = 0.900$, $\mu d_{d,33} = 3.00$, and $\mu d_{e,33} = 3.15$, with $\mu L \simeq 18$. We use the illustrative gauge-coupling unification value $g_i = g$ with $g^2/(4\pi) \simeq 0.04$, so that $a_u = 0.030$ and $a_d = a_e = 0.023$. Then $(\mu d_{u,33})^2 = 0.81$, which is smaller than the RHS of (9), viz., 46, while $(\mu d_{f,33})^2 >$ RHS (9) for $f = d, e$. Thus, with these input values, the Coulomb interactions cause t_L and t_L^c wavefunctions to be centered essentially on top of each other, but do not overwhelm the trapping of other fermions to their domain walls. Generalizing, we can say that if a fermion has $\mu d_{f,33} \lesssim O(1)$, i.e., is moderately heavy, then, for a value of $\mu L \sim 20$ that is phenomenologically acceptable, LHS (9) < RHS (9), so that the Coulomb interaction can dominate over the trapping interaction with Φ and cause the chiral components of this fermion to lie essentially on top of each other, leading to $m_f \sim M_{\mathrm{ew}}$ if $\kappa_{f,33} \sim O(1)$. For plausible input values, this can happen for a charge 2/3 quark while the charge $-1/3$ quarks and leptons remain trapped on their domain walls. This could thus help to explain the fact that the heaviest known fermion, and the only fermion with a mass comparable to the EWSB scale M_{ew}, is a charge 2/3 quark, rather than a charge $-1/3$ quark or a charged lepton and thus could provide deeper insight into properties (i) and (ii) in the abstract.

Although we have discussed this Coulomb-induced collapse in models with large extra dimensions in which standard model fields propagate, we also note that, more generally, gauge interactions could also be relevant to models with dynamical EWSB involving multifermion operators in which a $\langle \bar{f} f\rangle$ condensate forms. We suggest that these gauge interactions and the inequality (8) could explain why in such dynamical EWSB models it is the $\langle \bar{t} t\rangle$ condensate that forms rather than other condensates which, a priori, could form, such as $\langle \bar{b} b\rangle$ or $\langle \bar{\tau}\tau\rangle$.

Just as a WKB approximation can be used to infer the result (3) before inclusion of the Coulomb effects, so also it can be used to calculate the latter effects. We next do this for fermions for which this attraction does not overwhelm the trapping to domain walls. As in the derivation of (3), one can picture the physics in terms of a quantum-mechanical tunnelling process. Consider the symmetric path where f_i and f_j^c each tunnel a distance r toward each other from their respective centers (x, ℓ_{f_i}) and $(x, \ell_{f_j^c})$ so that they are a distance $2\epsilon_{f,ij} = d_{f,ij} - 2r$ apart. The classical turning points $(r_t)_{f,ij}$ occur where the total potential energy

$$V_{\mathrm{tot}}(r) = 2\mu^2 r - \frac{a_f L^n}{(d_{f,ij} - 2r)^{n+1}} \tag{10}$$

vanishes. We have $\epsilon_{f,ij} = (1/2)d_{f,ij} - (r_t)_{f,ij}$ and define $\eta_{f,ij} = 2\epsilon_{f,ij}/d_{f,ij}$. Then the equation for the turning point becomes

$$\eta^{n+1}_{f,ij}(1 - \eta_{f,ij}) = b_f, \tag{11}$$

where $b_f = a_f(\mu d_{f,ij})^{-2}(L/d_{f,ij})^n$. Since the mutual tunnelling by the fermions does not have to proceed further than to the midpoint between them, we are only interested in the range $0 \leqslant \eta_{f,ij} \leqslant 1$. For this range, the LHS of (11) increases from 0 to a maximum value $b_{f,m} = (n+1)^{n+1}/(n+2)^{n+2}$ at $(\eta_{f,ij})_m = (n+1)/(n+2)$ and then decreases to 0 again at $\eta_{f,ij} = 1$. If $b_f \leqslant b_{f,m}$, (11) has physical solutions. For $b_f = b_{f,m}$, there is a unique physical solution, $\eta_{f,ij} = (\eta_{f,ij})_m$. For $0 < b_f < b_{f,\max}$, the two turning points are given by $r^{(1,2)}_{f,ij} = (1 - \eta^{(2,1)}_{f,ij})d_{f,ij}/2$ with $0 \leqslant \eta^{(1)}_{f,ij} < (n+1)/(n+2) < \eta^{(2)}_{f,ij} < 1$. The relation between the higher-dimensional and 4D Yukawa couplings is then given by

$$\kappa^{(4D)}_{f,ij} = w_{f,ij}\,\kappa_{f,ij} \tag{12}$$

where the overlap factor is, in the WKB approximation, $w_{f,ij} = \exp(-J_{f,ij})$, with

$$J_{f,ij} = \int_{r^{(1)}_{f,ij}}^{r^{(2)}_{f,ij}} V_{\rm tot}(r)\,dr$$
$$= \left(\eta^{(2)}_{f,ij} - \eta^{(1)}_{f,ij}\right)\left[\left(\frac{\mu d_{f,ij}}{2}\right)^2 \left[2 - \left(\eta^{(1)}_{f,ij} + \eta^{(2)}_{f,ij}\right)\right] - \frac{a_f}{2n\eta^{(1)}_{f,ij}\eta^{(2)}_{f,ij}}\left(\frac{L}{d_{f,ij}}\right)^n\right]. \tag{13}$$

The integral extends over the classically forbidden region between the two turning points [7].[3] For the $n = 2$ parameters above, we find that the gauge interaction increases the overlap factors $w_{d,33}$ and $w_{e,33}$ relevant for m_b and m_τ by 22% from 0.77 to 0.98 and from 0.76 to 0.93, respectively. Thus, gauge interactions have a significant enhancement effect on the wavefunction overlaps and hence Yukawa couplings. Although we have concentrated on the case $g_3 = g_1$, the more general case $g_3 > g_1$ would lead to further enhancement of the masses of the charge $2/3$ and $-1/3$ quarks relative to those of the charged leptons. Of course, this is not an *ab initio* calculation of the fermion mass spectrum, since it depends on initial inputs for the relative distances $d_{f,ij}$. What our calculations show is that gauge interactions, together with the inequality (8), could help to explain why the quarks of a given generation are heavier than the charged lepton and why, at least for the higher two generations, $m_{Q=2/3} > m_{Q=-1/3}$, i.e., property (iii).

For the first generation, given the smallness of m_u and m_d,[4] a model for these masses should take account of the off-diagonal quantities $\kappa^{(4\rm D)}_{f,ij}$, $ij = 12, 21$. Indeed, if the $N_g = 1, 2$ subsectors of the mass matrices for the charge $2/3$ and $-1/3$ quarks have the form (after allowed rephasings so that $A^{(f)}_{22}$ is real and positive)

$$M^{(f)} = \begin{pmatrix} \sim 0 & A^{(f)}_{12} \\ A^{(f)}_{21} & A^{(f)}_{22} \end{pmatrix} \tag{14}$$

where $|A^{(f)}_{ij}|/A^{(f)}_{22} \ll 1$ for $ij = 12, 21$ and ~ 0 means a negligibly small entry, the eigenvalues have the form $\lambda^{(f)}_2 \simeq A^{(f)}_{22}$ and $|\lambda^{(f)}_1| \simeq |A^{(f)}_{12}A^{(f)}_{21}|/A^{(f)}_{22}$ (with $\lambda^{(u)}_2 = m_c$, $\lambda^{(d)}_2 = m_s$, $|\lambda^{(u)}_1| = m_u$, $|\lambda^{(d)}_1| = m_d$). Since $A^{(u)}_{22} \simeq m_c \gg A^{(d)}_{22} \simeq m_s$ and since these enter in the denominators of the expressions for the lighter eigenvalues, if $A^{(u)}_{ij}$ and $A^{(d)}_{ij}$, $ij = 12, 21$ are not too different, this seesaw effect could accomodate the fact that $m_u < m_d$. That is, these lightest quark masses could arise primarily by mixing, and hence could avoid the generic pattern $m_{u_i} > m_{d_i} > m_{e_i}$ for the heavier two generations $i = 2, 3$.

An important comment concerns the calculability of these gauge interaction effects. The fact that fermions with stronger gauge interactions (color and $U(1)_Y$ or $U(1)_{\rm em}$) are more massive is very suggestive. Yet conventional attempts to explain this via radiative corrections in usual quantum field theory encounter the obstacle that perturbative gauge boson couplings preserve chirality and cannot generate masses from originally massless fermions. Once fermion masses are generated, by the Higgs mechanism or in some other way, radiative gauge boson corrections modify the tree-level masses; however, these corrections are divergent, so that the physical masses are arbitrary numbers which are inserted to fit experiment. In contrast, our effects are calculable and finite. Even in the case where they overwhelm the domain-wall trapping, and hence there can be sensitivity to Λ, this just results in $w_{f,ij} \simeq 1$.

Some of the features that we have found may well transcend the specific low-energy effective field theory approach used here. Indeed, this approach leaves a number of open questions. What are the implications of the fact that the theory is nonrenormalizable in $d > 4$ dimensions? What physics gives rise to the restriction of the gauge and matter fields to $M \times [0, L]^n$, where M is Minkowski space, and to the localization of the fermion wavefunctions? The latter problem may well be related to a possible underlying type I string/brane theory. We speculate that the localization of fermions and resultant Gaussian profile might be achieved without introducing the Φ field and higher-dimensional Yukawa couplings and masses. Recall that in the type I approach, the open strings end on D-branes (while the closed strings required for unitarity can propagate in the bulk), and, via the asso-

[3] We note the similarities between our present calculation and the use of the WKB approximation in estimating, e.g., the quantum-mechanical tunnelling through a Coulomb barrier in the case of α decay of nuclei.

[4] We refer to the running quark masses evaluated at a common scale $\gg \Lambda_{\rm QCD}$.

ciation of Chan–Paton factors with the ends of these strings, coincident D-branes give rise to gauge symmetries such as $U(N)$ [8,9]. Separating these branes corresponds to the breaking of these gauge symmetries or, in the T-dual view, to the appearance of Wilson lines. The breaking of translation invariance in the directions orthogonal to the D-branes has, as its T-dual manifestation, the nonconservation of the open string winding number w, corresponding to the fact that the string can break [9]. Given the association of the Chan–Paton factor with the end of the open string, it follows that a free end with index i transforms as a fundamental representation of the gauge group. As is clear from the 't Hooft double-line representation $i\bar{j}$ for a $U(N)$ gauge boson, this string breaking is reminiscent of the field-theoretic vertex in which a gauge boson creates a fermion–antifermion pair. Further, recall that a string generates a linear confining potential; this, in turn, gives rise to a Gaussian amplitude $\propto e^{-Ku^2}$ (where K is proportional to the string tension) for having a string extending a length u in the extra compact dimensions. This suggests that in this framework one could interpret the fermions as half-strings with mixed Dirichlet–Neumann boundary conditions. This is very suggestive since it completes in a natural way the correspondence between various geometrical structures and elementary light particles of ascending spins. Thus, as in lattice gauge theories, the elementary spin-zero scalar fields are identified with points on the lattice or here on the branes. The (x, t)-averaged locations of the branes and their separations are related to Higgs vevs, as in the picture of [8,9]. Strings, or correspondingly elementary links, correspond to the gauge bosons, and the more complex closed-strings, somewhat analogous to plaquettes in lattice gauge theory, correspond here to the spin-two gravitons. Thus, the half strings in this hierarchy are intermediate between points and links (Dirichlet strings ending on two D-branes), a very suggestive relation. As indicated, this may help to explain the fermion localization on branes.

In summary, we have shown that the inclusion of Coulombic gauge interactions in models with fermion wavefunctions separated in extra dimensions has important consequences and could help to provide an explanation of several of the most basic features of the known fermion masses. This explanation is minimal in that it makes use only of the established gauge transformation properties of the known fermions, albeit in a new context. Clearly, of course, the ultimate viability of this explanation requires that experimental evidence be found for large extra dimensions of the type considered here, in which standard model fields propagate.

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